Optimal Asset Selection for Securitization Structures

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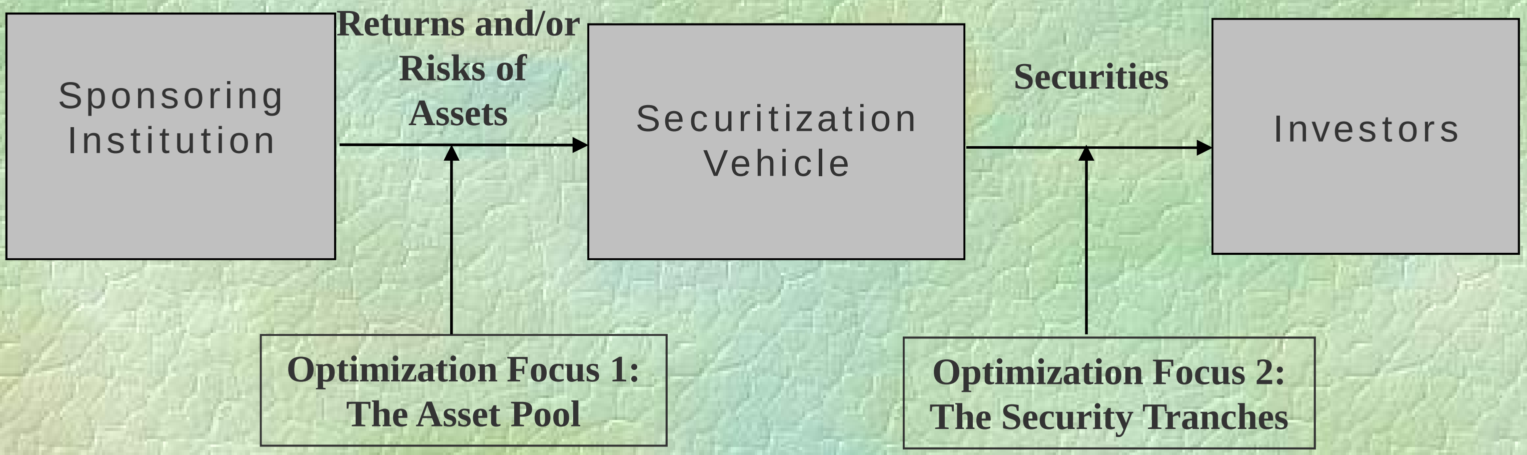
Table of Contents

Introduction

Securitization Process Overview



Sponsoring Institution
Returns and/or Risks of Assets
Securitization Vehicle
Securities
Investors
Optimization Focus 1: The Asset Pool
Optimization Focus 2: The Security Tranches

In cash or balance sheet securitizations asset risks and returns are converted into Notes and Equity securities. In addition, money is posted by the institution for a Cash Collateral Account.

In a synthetic securitization, the default driven loss (not credit degradation) risks of assets are converted into Notes, Equity and a First Loss position which is retained by the institution.

Introduction

Goals of Securitization

1. **Funding Source**
2. **Balance Sheet Reduction**
3. **Regulatory Capital Relief**
4. **Risk Reduction**

Introduction

Challenges to Completion

1. Expense to Institution
2. Market Appetite for Securities
3. Rating Agency Requirements
4. Regulatory Requirements

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Introduction

Goal of Asset Selection

Create the optimal pool of assets that meets the institution’s goals in terms of funding relief, balance sheet reduction, regulatory relief and risk reduction while overcoming the challenges of cost, rating agency, market and regulatory constraints.

Table of Contents

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Optimization

Purpose

•For a given problem with a variety of potential solutions we wish to accomplish two contradictory things

1. Explore as many of the potential solutions as possible

ex. Random Search

2. Exploit the information in the best solutions

ex. Hillclimbing

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Optimization

Examples of Techniques

•**<u>Hillclimbing</u> or Steepest Ascent uses a numerical derivative for an indication of the direction to the nearest maximum solution to a problem in one or multiple dimensions**

•**<u>Simulated Annealing</u> creates a physical analog to the process of slowly cooling a liquid to a solid and takes advantage of the very low energy level of the resulting crystal structure to minimize a target function or problem**

•**<u>Genetic Algorithm</u> or Evolution Program creates a biological analog to the process of evolution where potential solutions vie against other potential solutions to pass their information to the next generation of solutions**

Optimization

Kangaroo hopping to find Mt. Everest analogies - see Michalewicz reference

- **Hillclimbing**: The kangaroo can at best hope to find the top of the mountain closest to where he starts. There's no guarantee that this mountain will be Everest, or even a very high mountain.

- **Simulated Annealing**: The kangaroo is drunk and hops around randomly for a long time. However, he gradually sobers up and tends to hop up hill.

- **Genetic Algorithm**: Lots of kangaroos are parachuted into the Himalayas at random places. These kangaroos do not know what they are looking for. However every few years you shoot the kangaroos at low altitudes and hope the ones that are left will be fruitful and multiply.

Optimization

Why use a genetic algorithm?

- **Large pool of assets to choose from**
- **Many constraints both hard and soft**
- **Tremendous flexibility in meeting multiple goals**
- **Inclusion or exclusion of an asset from a pool is a binary decision and fits the analogy perfectly**
- **It works**

Table of Contents

Genetic Algorithm - the Basics

Definitions

- **Generation - a collection of potential solutions to a problem**
- **Fitness - a measure of how good a proposed solution is**
- **Selective Reproduction - choosing two proposed solutions as "parents" to a new solution based upon their fitness measures**
- **Diversity - a means of sharing descriptive information from parents in the creation of a new solution. Composed of cross-over and mutation**

Genetic Algorithm - the Basics

Generation of Potential Portfolios

How do we describe a potential portfolio in terms of the n assets the institution has available for inclusion?

We create a string of 0s and 1s. Where each position refers to a potential asset and each asset is either included (=1) or excluded (=0) from the portfolio.

	Asset 1	Asset 2	Asset 3	Asset 4	…..….	Asset n
Portfolio 1 (Generation x):	0	1	0	1	……..	1
Portfolio 2 (Generation x):	1	1	0	0	……..	1
Portfolio 3 (Generation x):	0	0	0	0	……..	1
⋮						
Portfolio y (Generation x):	1	1	0	1	……..	0

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Fitness

Given the institution’s goals for pursuing the securitization, the degree to which the portfolio meets those goals results in a positive contribution toward fitness.

For each constraint on the transaction, a penalty is assigned. The degree to which each constraint is broken is multiplied by the penalty. The sum of the penalties results in a negative contribution toward fitness.

Portfolio Fitness = Positive contributions - Penalties for Exceeding Constraints

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Genetic Algorithm - the Basics

Selective Reproduction

For each generation of portfolios the fitness of each portfolio is used for determining whether it is chosen as a parent for the creation of a child in the next generation. In this example, we wish to maximize regulatory capital relief where RWA = Risk Weighted Assets.

Generation 7 (500 Portfolios)				
	RWA	Penalties	Fitness	Likelihood
Portfolio 008	$372,000,000	$109,000,000	$263,000,000	1.32%
Portfolio 092	$383,000,000	$123,000,000	$260,000,000	1.30%
Portfolio 119	$302,000,000	$108,000,000	$194,000,000	0.97%
Portfolio 313	$312,000,000	$123,000,000	$189,000,000	0.95%
Portfolio 101	$299,000,000	$112,000,000	$187,000,000	0.94%
Portfolio 149	$329,000,000	$165,000,000	$164,000,000	0.82%
Portfolio 012	$234,000,000	$99,000,000	$135,000,000	0.68%
Portfolio 415	$234,000,000	$123,000,000	$111,000,000	0.56%

Various sampling methods are available using either the relative level of fitness as compared to total generation fitness (assuming generation fitness total of $20 Billion as above) or ordinal ranking methods based absolutely on position.

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Genetic Algorithm - the Basics

Diversity 1: Crossover

Having selected two parent portfolios, how do we combine them to create a new child portfolio?

We assign a likelihood of cross-over and having randomly drawn for it, a random point of occurrence.

CROSSOVER:

Point of Cross-Over

Asset 1 Asset 2 Asset 3 Asset 4…..…

Parent 1 (Generation 7, Portfolio 213): 001010011 0010101100011011 10000…………

Parent 2 (Generation 7, Portfolio 107): 111000101 011000111000011010100…………

Resulting Child (Generation 8, Portfolio 1): 001010011 011000111000011010100…………

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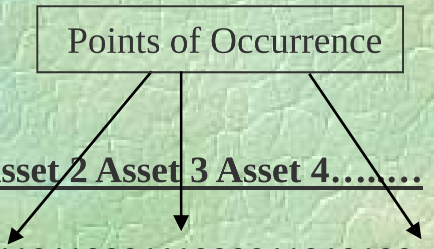
Genetic Algorithm - the Basics

Diversity 2: Mutation

Having created a child portfolio, we assign a likelihood of mutation for each new bit position and having randomly drawn for it, a mutation occurs. Mutation entails flipping a one to a zero and vice-versa.

MUTATION:



<u>Asset 1 Asset 2 Asset 3 Asset 4……..</u>

Child Before (Generation 8, Portfolio 1): <u>00101001101100011100001101010 0…………</u>

Child After (Generation 8, Portfolio 1): <u>001010001011000101000011110101…………</u>

Genetic Algorithm - the Basics

The Resulting Process

1. **Randomly create a collection of portfolios (the first generation) that each produce a positive fitness score subject to constraints.**

2. **Calculate and rank order the fitness of each one of these portfolios**

3. **Select parents (portfolios) based upon the fitness level of each portfolio as a percentage of total fitness.**

4. **Use cross-over and mutation to combine selected parents into a new generation of portfolios.**

5. **Repeat steps 2 through 4 a desired number of times and store the best 10 results over the life of the process. By no means will the best portfolio be in the last generation (no explicit convergence)**

Table of Contents

Genetic Algorithm - Advanced

Fitness Scaling

The natural tradeoff between covering the state space as well as using information in the best solutions can lead to two problems.

1. Outstanding early results (optimal portfolios) can dominate future generations.

2. Moderately improved later results (optimal portfolios) are not dominant enough.

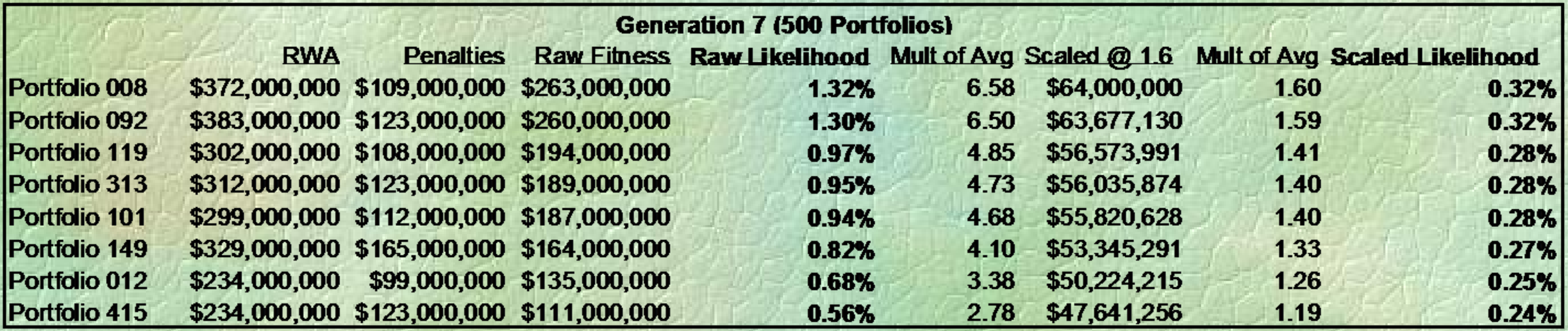
Fitness Scaling is a means to overcome these problems. We desire a fixed multiple between the fitness of the best solution and the average fitness of all solutions in any given generation.

Typically set to between 1.5 and 2.0 multiplied by the average fitness, this restrains the selective reproduction of the algorithm in the initial generations (where dispersion is large) and enhances it in later generations (where dispersion is small)

Genetic Algorithm - Advanced

Scaled Fitness - Example

In this example, a linear scaling constant of 1.6 is applied to the same exhibit presented earlier. Again, the generation total raw fitness amount is $20 Billion producing an average raw fitness level of $40 million per portfolio.



Generation 7 (500 Portfolios)

	RWA	Penalties	Raw Fitness	Raw Likelihood	Mult of Avg	Scaled @ 1.6	Mult of Avg	Scaled Likelihood
Portfolio 008	$372,000,000	$109,000,000	$263,000,000	1.32%	6.58	$64,000,000	1.60	0.32%
Portfolio 092	$383,000,000	$123,000,000	$260,000,000	1.30%	6.50	$63,677,130	1.59	0.32%
Portfolio 119	$302,000,000	$108,000,000	$194,000,000	0.97%	4.85	$56,573,991	1.41	0.28%
Portfolio 313	$312,000,000	$123,000,000	$189,000,000	0.95%	4.73	$56,035,874	1.40	0.28%
Portfolio 101	$299,000,000	$112,000,000	$187,000,000	0.94%	4.68	$55,820,628	1.40	0.28%
Portfolio 149	$329,000,000	$165,000,000	$164,000,000	0.82%	4.10	$53,345,291	1.33	0.27%
Portfolio 012	$234,000,000	$99,000,000	$135,000,000	0.68%	3.38	$50,224,215	1.26	0.25%
Portfolio 415	$234,000,000	$123,000,000	$111,000,000	0.56%	2.78	$47,641,256	1.19	0.24%

Note that the high likelihood of being selected as a parent for portfolios 8 and 92 is reduced by 75%. In generations far into the algorithm the result is an increased likelihood of selection.

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Scaled Penalties

The number and scope of the penalty functions for meeting all transaction constraints results in one of two scenarios:

1. The final generation has not seen any portfolios in any generation that meet all constraints

2. It is impossible to produce any random portfolios in the first generation where the fitness measure is not negative. As negative fitness measure portfolios cannot be scaled and are thus eliminated (executed?) at creation, no solutions are found.

The answer is scaled penalties.

The final penalties are assigned and reduced by a penalty scalar at the start of the run. This scalar is gradually incremented to one over the course of a set number of generations.

For example, begin by applying 2% of the penalty amount in the first generation, and increment by 2% each generation until the full penalty is applied beginning in generation 50

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Multiple Cross-Over Points

The primary information mixing source between selected parents is crossover. By using multiple cross-over points, mixing proceeds more quickly and thoroughly (technically, more schema patterns can be produced with multiple cross-over points see references)

Two Crossover Points:

Points of Cross-Over

Asset 1 Asset 2 Asset 3 Asset 4……..

Parent 1 (Generation 7, Portfolio 213): 001010011 0010101100011011 10000…………

Parent 2 (Generation 7, Portfolio 107): 111000101 0110001110000110 10100…………

Resulting Child (Generation 8, Portfolio 1): 001010011 0110001110000110 10000…………

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Elitism

As mentioned, convergence is not a property of genetic algorithms. Generally the best resulting solution does not occur in the last generation. If it does, one can conclude more generations could be run (depending upon computational time limits).

Actual results of an 800 generation, 400 member run:

Generation	Member	Fitness
371	369	$939,438,737
506	45	$932,753,580
495	90	$905,020,940
620	27	$899,417,690
476	201	$888,512,928
559	212	$882,347,651
595	81	$880,619,911
334	275	$878,520,755
396	230	$877,481,553
691	148	$875,479,717

Genetic Algorithm - Advanced

Elitism (continued)

The best solution may or may not pass its information to the next generation given the random draws in creating children.

Elitism is a way to ensure the best member is among the children.

Typically, at the beginning of the creation of each succeeding generation, before any parents are selected for the creation of children, the best solution (portfolio) is cloned and placed as the first child of the succeeding generation.

In this manner, even if it is not chosen randomly, its information (genes) is maintained in the population for later selection of ensuing generations.

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Partial Assets

The entire method as presented so far entails choosing or excluding an asset from consideration absolutely. However, some constraints, such as maximum obligor amounts will completely exclude many assets. A means of assigning partial assets requires pursuing one of two paths:

1. a method consistent with the binary cross-over and mutation operators

2. a new way of defining cross-over and mutation to work with decimal numbers

Most fundamental research indicates that choice 1 should provide the best solutions. However, many implementations of choice 2 have provided good solutions.

We will pursue choice 1.

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Partial Assets (continued)

We will need to assign multiple bits to each asset. Then the value from these bits can be converted to a percentage. Given a minimum asset increment (ex. $1 million) the percentage can be applied to the available asset with rounding to provide a final asset amount for consideration in the fitness measure.

Assuming 6 bits per Asset:

	Asset 1	Asset 2	Asset 3	Asset 4……..…	Asset n
Portfolio 1 (Generation x):	010110	110011	100001	000001……..	010111

1. For asset 1: $0 * 2^0 = 00$
$1 * 2^1 = 02$
$1 * 2^2 = 04$
$0 * 2^3 = 00$
$1 * 2^4 = 16$
$0 * 2^5 = \underline{00}$
22

2. Max Value is $2^{\text{Bits per Facility}} - 1 = 63$

Resulting Percentage = 22/63 = 35%

For $50 million asset, assign $17 million

All other operations work exactly as before on a much longer string describing the assets. Computational burden increases significantly.

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Table of Contents

Practical Issues and Next Steps

Data

- Screening source information and feeding model
- Asset Descriptions
    - Issuer
    - Industry
    - Seniority/Security
- Before/After Transaction
    - Partial Assignment
    - Pro-rata vs. First Drawn
    - Multiple Transactions

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Practical Issues and Next Steps

Risk Measurement

- **Defining Risk**
  - **default vs. deterioration**
  - **recovery**
- **Rating Agency**
  - **historical vs. stress**
- **Internal**
- **External**

Practical Issues and Next Steps

Tenor

- **Modeling the pool vs. the assets**
  - **Extensions to transaction**
  - **Asset roll-over assumptions**
- **Rating Agency Issues**
- **Regulatory Issues**

Practical Issues and Next Steps

Extending the Model

- **Supporting Multiple Transactions**
- **Managing asset additions**
- **Optimization Focus 2: Structuring Tranches of Securities**
    - **Market spread levels**
    - **Market appetite for amounts**
    - **Rating agency subordination requirements and stress tests**

Practical Issues and Next Steps

References

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•Genetic Algorithms In Search, Optimization & Machine Learning, (1989) by David E. Goldberg

•Genetic Algorithms + Data Structures = Evolution Programs, 3d Edition (1996) by Zbigniew Michalewicz